



Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE Further Mathematics

AS Further Statistics 1 Paper 8FM0_23

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 40.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol \surd will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - \square The second mark is dependent on gaining the first mark
4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
 5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.
 6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Qu	Scheme	Marks	AO
1(a)	H_0 : There is no association between age and activity H_1 : There is an association between age and activity	B1 (1)	1.2
(b)	$\frac{26 \times 36}{150} = 6.24$	B1 (1)	1.1b
(c)	Since expected value in $a < 20$ and snooker = $3.6 < 5$ we amalgamate two rows Table is now 4×3 so degrees of freedom is $(4-1) \times (3-1) = 6$ (*)	B1 B1*	2.4 1.1b
(d)	Critical value $\chi_6^2(5\%) = 12.592$ [Significant result]: so there is evidence to support manager's belief	B1 B1ft (2)	1.1b 2.2b
		(6 marks)	

Notes

(a)	B1 for both hypotheses in terms of "association" or "independence". Must mention age and activity (or sport). [Use of "relationship" or "link" here is B0 but allow for last B1ft]
(b)	B1 for 6.24
(c)	1 st B1 for a reason to get a 4 x 3 table based on amalgamation of rows Must mention $a < 20$ and snooker and see 3.6 and be combining <u>rows</u> (not columns) 2 nd B1* for 6 degrees of freedom clearly coming from a 4 x 3 table formed from amalgamation of <u>rows</u> . [$8 - 2 = 6$ is B0]
(d)	1 st B1 for correct critical value (allow 12.6 or 12.59 or awrt 12.592) NB p -value = 0.0032839... so allow awrt 0.00328 2 nd B1ft for a correct comparison and conclusion (ft their cv) [<u>Independent of hypotheses</u>] e.g. there is an "association" or "relationship" or "link" between age and activity is OK BUT there is a "correlation" between age and activity is B0 Do not accept contradictory contextual statements e.g. "manager's belief supported , there is no association between age and activity"

Qu	Scheme	Marks	AO																								
2.	H ₀ : Spinner is working as designed (o.e.) H ₁ : Spinner is not working as designed (o.e.)	B1	1.2																								
	<table border="1"> <tr> <td>E_i</td> <td>24</td> <td>8</td> <td>8</td> <td>32</td> <td>8</td> </tr> <tr> <td>O_i</td> <td>15</td> <td>4</td> <td>12</td> <td>41</td> <td>8</td> </tr> <tr> <td>$\frac{(O_i - E_i)^2}{E_i}$</td> <td>$\frac{81}{24}$</td> <td>$\frac{16}{8}$</td> <td>$\frac{16}{8}$</td> <td>$\frac{81}{32}$</td> <td>0</td> </tr> <tr> <td>$\frac{O_i^2}{E_i}$</td> <td>$\frac{225}{24}$</td> <td>$\frac{16}{8}$</td> <td>$\frac{144}{8}$</td> <td>$\frac{1681}{32}$</td> <td>$\frac{64}{8}$</td> </tr> </table>	E_i	24	8	8	32	8	O_i	15	4	12	41	8	$\frac{(O_i - E_i)^2}{E_i}$	$\frac{81}{24}$	$\frac{16}{8}$	$\frac{16}{8}$	$\frac{81}{32}$	0	$\frac{O_i^2}{E_i}$	$\frac{225}{24}$	$\frac{16}{8}$	$\frac{144}{8}$	$\frac{1681}{32}$	$\frac{64}{8}$	M1 A1	3.4 1.1b
	E_i	24	8	8	32	8																					
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	$\sum \frac{(O_i - E_i)^2}{E_i} = 3.375 + 2 + 2 + 2.53125 + 0 = 9.90625$	M1	1.1b																								
$\text{or } \sum \frac{O_i^2}{E_i} - N = 9.375 + 2 + 18 + 52.53125 + 8 - 80 = 9.90625$	A1	1.1b																									
$v = 5 - 1 = 4$ so $\chi_4^2(10\%)$ cv = 7.779 or better	B1	3.4																									
Result is significant so there is evidence that the spinner is not operating as designed	A1cso	3.5a																									
		(7 marks)																									
Notes																											
1 st B1 for both hypotheses given in suitable context 1 st M1 for using the model to find at least 2 correct expected frequencies 1 st A1 for all correct E_i 2 nd M1 for attempt to find test statistic (at least two correct expressions, fractions or decimals) 2 nd A1 for a correct test statistic (awrt 9.91) [accept $\frac{317}{32}$] 2 nd B1 for correct critical value (allow 7.78) NB p - value = 0.042036...so allow awrt 0.042 3 rd A1cso dep on <u>all</u> previous marks for a correct conclusion in context (can be in terms of model or spinner's design) Must mention spinner and scores <u>or</u> design Accept "spinner is not accurate"																											

Qu	Scheme	Marks	Alt
3(a)	[X = number of errors in 100-word piece] $X \sim \text{Po}(1.7)$	M1	3.3
	$P(X < 2) = P(X \leq 1) = 0.49324\dots$ awrt <u>0.493</u>	A1	1.1b
		(2)	
(b)	[R = number of errors in the article] $R \sim \text{Po}(4.25)$	M1	3.3
	$P(R = 5) = 0.16482\dots$ awrt <u>0.165</u>	A1	1.1b
		(2)	
(c)	Scheme A : Let $A \sim B(40, e^{-1.7})$ or $B(40, 0.18268\dots)$	M1	3.3
	$P(A > 10) = 1 - P(A \leq 10)$	M1	1.1b
	$= 0.0995591\dots$ awrt <u>0.0996</u>	A1	1.1b
	Scheme B : Let $B \sim \text{Po}(40 \times 1.7)$ or $\text{Po}(68)$	M1	3.3
	$P(B < 56) = P(B \leq 55) = 0.061133\dots$		
	So choose scheme A (since the probability of a bonus is greater)	A1	2.4
		(5)	
(d)	$H_0: \lambda = 1.7$ (or $\mu = 8.5$) $H_1: \lambda < 1.7$ (or $\mu < 8.5$)	B1	2.5
	[E = no. of errors in the piece of work] $E \sim \text{Po}(8.5)$	M1	3.3
	$P(E \leq 3) = 0.0301$ <u>or</u> $P(E \leq 4) = 0.0744$	A1	1.1b
	So critical region is $E \leq 3$	A1	2.2a
			(4)
(13 marks)			

Notes

(a)	M1 for selecting the correct Poisson distribution A1 for awrt 0.493
(b)	M1 for selecting the correct Poisson distribution A1 for awrt 0.165
(c)	1 st M1 for choosing a correct model for scheme A i.e. $B(40, P(X = 0))$, where $X \sim \text{Po}(1.7)$ Allow use of awrt 0.183 for $P(X = 0) \dots 0.183$ gives answer awrt 0.101 Condone $B(0.183, 40)$ (o.e.) if it leads to a prob rounding to range (0.09~0.1) otherwise M0 2 nd M1 for $1 - P(A \leq 10)$ 1 st A1 for awrt 0.0996 [NB use of 0.183 will give awrt 0.101 and scores M1M1A0] 3 rd M1 for selecting a correct Poisson model for scheme B i.e. $\text{Po}(40 \times 1.7)$ or better 2 nd A1 for a correct conclusion based on comparing two probs: awrt 0.1 vs 0.061 or better So can allow $0.1 > 0.061$ leading to choosing A [Probably scores M1M1A0M1A1]
NB	[Normal approx.(not on spec) leading to 0.06477... might score 3 rd M1 if $\text{Po}(68)$ seen but 2 nd A0]
(d)	B1 for both hypotheses in terms of λ or μ (can be interchanged) M1 for selecting $\text{Po}(8.5)$ (sight of or use of e.g. may be implied by 1 st A1) 1 st A1 for some evidence of correct use of $\text{Po}(8.5)$ i.e. either of these probs (2dp or better) May be implied by a correct critical region 2 nd A1 for a correct critical region. Allow $E < 4$ and allow any letter for E . <u>Two</u> different regions (e.g. from 2 tail test) is 2 nd A0
SC	Use of binomial throughout: (with hypotheses $H_0: p = 0.017$ and $H_1: p < 0.017$ in (d)) Scores 0 in (a) 0 in (b) possibly just 2 nd M1 in (c) But allow all 4 marks in (d): B1 hypotheses, M1 for $Y \sim B(500, 0.017)$, 1 st A1 for $P(Y \leq 3) = 0.02913\dots$ <u>or</u> $P(Y \leq 4) = 0.07266\dots$ 2 nd A1 $Y \leq 3$ Allow probs to be to 2dp or better so 0.03 and 0.07 as in main scheme.

Qu	Scheme	Marks	Alt																		
4(a)	$q + \frac{7}{30}$	B1 (1)	1.1b																		
(b)	$E(X^2) = (-3)^2 \times q + (-1)^2 \times \frac{7}{30} + 1^2 \times \frac{7}{30} + 2^2 \times q + 4^2 \times r$ $= \frac{7}{15} + 13q + 16r$ (*)	M1 A1*cs0 (2)	1.1b 1.1b																		
(c)	$E(X) = -3q + -\frac{7}{30} + \frac{7}{30} + 2q + 4r$ { = $4r - q$ } $E(X^2 + 6X) = \frac{7}{15} + 7q + 40r$ $E(X^3) = (-3)^3 \times q + (-1)^3 \times \frac{7}{30} + 1^3 \times \frac{7}{30} + 2^3 \times q + 4^3 \times r$ $= 64r - 19q$ Sum of probabilities = 1 gives: $2q + r = \frac{16}{30}$ (o.e.) Solve: $24r - 26q = \frac{7}{15}$ and $r + 2q = \frac{8}{15}$ e.g. $37r = \frac{111}{15}$ So $r = \frac{1}{5}$ and $q = \frac{1}{6}$	M1 A1 M1 A1 M1 dM1 A1 (7)	3.1a 1.1b 3.4 1.1b 1.1b 1.1b 1.1b																		
(d)	$X^3 > X^2 + 6X \Rightarrow X(X-3)(X+2) > 0$ Use of sketch or table to see: $-2 < X < 0$ or $X > 3$ So $P(X^3 > X^2 + 6X) = P(X = -1 \text{ or } 4)$ $= \frac{7}{30} + "r" = \frac{13}{30}$	M1 A1 M1 A1ft (4)	2.1 1.1b 2.2a 1.1b																		
ALT	<table border="1"> <tr> <td>X</td> <td>-3</td> <td>-1</td> <td>1</td> <td>2</td> <td>4</td> </tr> <tr> <td>X^3</td> <td>-27</td> <td>-1</td> <td>1</td> <td>8</td> <td>64</td> </tr> <tr> <td>$X^2 + 6X$</td> <td>-9</td> <td>-5</td> <td>7</td> <td>16</td> <td>40</td> </tr> </table>	X	-3	-1	1	2	4	X^3	-27	-1	1	8	64	$X^2 + 6X$	-9	-5	7	16	40	(14 marks)	
X	-3	-1	1	2	4																
X^3	-27	-1	1	8	64																
$X^2 + 6X$	-9	-5	7	16	40																

Notes

(b)	M1 for at least 3 correct terms of the expression for $E(X^2)$ A1*cs0 evidence of M1 scored with no incorrect working seen leading to correct answer (*) Allow $-3^2 \times q + -1^2 \times \frac{7}{30}$ etc if followed by $9q + \dots$ but <u>not</u> if simply followed by given answer
(c)	1 st M1 for realising the need to find $E(X)$ – a correct attempt with at least 3 correct terms 1 st A1 for the correct expression (needn't be simplified at this stage) 2 nd M1 for a correct attempt at $E(X^3)$ with at least 3 correct terms seen Treat no $\frac{7}{30}$ terms as <u>one</u> correct term 2 nd A1 for $64r - 19q$ (must be simplified) <u>or</u> for $24r - 26q = \frac{7}{15}$ 3 rd M1 for using sum of probabilities = 1 to form an equation in q and r (needn't be simplified) Must be correct or clearly state that $\Sigma\text{probs} = 1$ being attempted with only one slip 4 th dM1 for solving their 2 linear equations in q and r (dep on 3 rd M1 and 1 st <u>or</u> 2 nd M1) Must see correct method to reduce to a linear equation in one variable 3 rd A1 for $r = \frac{1}{5}$ <u>and</u> $q = \frac{1}{6}$ or any exact equivalents (dep on 2 correct equations seen)
(d)	1 st M1 for 1 st stage towards solving the inequality (factorising the cubic) 1 st A1 for solving the inequality 2 nd M1 for identifying the values of X required i.e. -1 and 4 2 nd A1ft for $\frac{13}{30}$ or exact equivalent e.g. 0.43 (Allow ft of "their r " + $\frac{7}{30}$)
ALT	Table 1 st M1 for at least 4 correct values for X^3 <u>and</u> $X^2 + 6X$ (must be labelled) 1 st A1 for all 10 correct values. [NB Can score M1A0M1A1ft in (d)]

